Leah Wrenn Berman
Representations of configurations

1 Individual Project's contribution to the CRP

1.1 Aims and Objectives

A *geometric* (q, k)*-configuration* is a collection of points and lines (or pseudolines) in the Euclidean or projective plane so that each point lies on q lines and each line passes through k points; if $q = k$, we merely refer to a k-configuration. Although 3-configurations have been studied since the late 1800s [16], it is only in the past thirty years that there has been significant study of more highly incident configurations, primarily of 4-configurations [10, 11, 12, 13, 14, 15, 17]. Many of the results, including most known infinite classes of configurations, arise from configurations with non-trivial geometric symmetries [8, 2, 4, 3, 6, 5, 7, 9].

Every configuration has a corresponding bipartite *Levi graph*, formed by associating a vertex of the graph to each point and line of the configuration and connecting two vertices of the graph with an edge precisely when the corresponding point and line are incident in the configuration.

In 2003, Boben and Pisanski [10] used *voltage graphs* to construct and analyze classes of highly symmetric configurations, including one of the most well-understood classes of 4-configurations, the *celestial/h-astral* 4-configurations.

However, little is known about more highly incident configurations—that is, (q, k)-configurations where one of q and k is more than 4; in particular, only one infinite class of 5-configurations [9] and one infinite class of 6-configurations are known [1].

The current project has two, related objectives. First, find new examples of symmetric k- and (q, k)-configurations, by constructing and analyzing associated graphs, such as Levi graphs or reduced Levi graphs/voltage graphs. For example, the known infinite class of 6-configurations has the Desargues graph as its reduced Levi graph. Do other small cubic graphs with high amounts of symmetry correspond to other classes of 6-configurations (possibly using pseudolines rather than lines)?

Second, is it possible to construct new classes of graphs, or to analyze known classes of graphs, by viewing them as Levi graphs or reduced Levi graphs of configurations? For example, the Levi graph of the Desargues configuration (which corresponds to Desargues' theorem from projective geometry) is a *cubic partial cube*, of interest in computer science. Do other configurations produce Levi graphs with interesting properties?

1.2 Methodologies

Potential avenues for investigation include:

- investigation of small examples, and construction of pseudoline examples
- construction of an algorithm to try to determine existence of configurations with a given voltage graph
- consideration of oriented matroids
- construction of the voltage graphs corresponding to known classes of configurations (e.g., floral, 5-configurations, 6-configurations)
- development of computer programs to construct or analyze configurations and associated graphs

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References

[1] Leah Wrenn Berman, *Constructing highly incident configurations*, In review.

[2] Leah Wrenn Berman, *Even astral configurations*, Electron. J. Combin. **11** (2004), no. 1, Research Paper 37, 23 pp. (electronic).

[3] Leah Wrenn Berman, *Movable* (n_4) *configurations*, Electron. J. Combin. **13** (2006), no. 1, Research Paper 104, 30 pp. (electronic).

[4] Leah Wrenn Berman, *Some results on odd astral configurations*, Electron. J. Combin. **13** (2006), no. 1, Research Paper 27, 31 pp. (electronic).

[5] Leah Wrenn Berman, *Astral* (n_4) *configurations of pseudolines*, Contrib. Discrete Math. **3** (2008), no. 2, 9–24.

[6] Leah Wrenn Berman, *A new class of movable* (n_4) *configurations*, Ars Math. Contemp. **1** (2008), no. 1, 44–50.

[7] Leah Wrenn Berman and Jürgen Bokowski, *Linear astral* (n_5) *configurations with dihedral symmetry*, European J. Combin. **29** (2008), no. 8, 1831–1842.

[8] Leah Wrenn Berman and Branko Grünbaum, *Deletion constructions of symmetric 4-configurations*, Contrib. Discrete Math. **5** (2010), no. 1.

[9] Leah Wrenn Berman and Laura Ng, *Constructing 5-configurations with chiral symmetry*, Electron. J. Combin. **17** (2010), no. 1, Research Paper 2, 14.

[10] M. Boben and T. Pisanski, *Polycyclic configurations*, European J. Combin. **24** (2003), no. 4, 431–457.

[11] Jürgen Bokowski, Branko Grünbaum, and Lars Schewe, *Topological configurations* (n_4) *exist for all* $n \geq 17$, European J. Combin. **30** (2009), no. 8, 1778–1785.

[12] Jürgen Bokowski and Lars Schewe, *There are no realizable* 15_4*- and* 16_4*-configurations*, Rev. Roumaine Math. Pures Appl. **50** (2005), no. 5-6, 483–493.

[13] Branko Grünbaum, *Connected* (n_4) *configurations exist for almost all* n, Geombinatorics **10** (2000), no. 1, 24–29.

[14] Leah Wrenn Berman, *Connected* (n_4) *configurations exist for almost all* n*—an update*, Geombinatorics **12** (2002), no. 1, 15–23.

[15] Leah Wrenn Berman, *Connected* (n_4) *configurations exist for almost all* n*—second update*, Geombinatorics **16** (2006), no. 2, 254–261.

[16] Leah Wrenn Berman, *Configurations of points and lines*, Graduate Studies in Mathematics, vol. 103, American Mathematical Society, Providence, RI, 2009.

[17] Branko Grünbaum and J. F. Rigby, *The real configuration* (21_4), J. London Math. Soc. (2) **41** (1990), no. 2, 336–346.