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Near Products

1 Individual Project's contribution to the CRP

1.1 Aims and Objectives

The visualization of graphs and networks, that is, of relational structures, needs specific methods that depend on the type of the graph to be visualized and on the purpose of the visualization. We are concerned with graphs that are products, graph bundles, or coverings of graphs and graphs that can be approximated by such structures.

Roughly speaking, given a graph G, the aim is to provide several, usually smaller graphs, and a rule for the construction of a graph H that is either identical with G or *near* to G. This can then be used to provide a visual presentation of the entire structure or a visual presentation of one or more homomorphic images of H that capture essential features of G.

The overall objective is the provision of algorithms that yield the desired structural information. This includes the recognition of graph products, bundles and coverings. Whereas the recognition of graph products is well advanced, the recognition of strong and direct products is still polynomial of high complexity and improvements are needed.

If a graph is not a product, but reasonably close to it (small edit distance), it is often called approximate. For the recognition of approximate graph products several algorithms have been proposed and tested, see [2, 5]. Nonetheless, improvements are needed here too.

If a graph originally had a product structure, but if its representation was significantly disturbed, different methods are needed. One might think of imposing a product structure on the given graph, be it for visualization or for the recognition of hidden structural properties. Graphs that we will represent in such a way will still be called *near products.*

Since algorithms for the representation of near products will usually return many different products the need arises to evaluate them.

In short, the aim of the IP encompasses the **provision of one or more product structures to a given graph** that capture essential features and which **can be used to render visualizations** of the given structure, of parts of it, or of superstructures. It may also include a reason **why a certain product was chosen** and an **estimate of the quality** of the representation.

The objectives of the IP include a **set of algorithms that provide the structures** of interest. It is intended to write them in C/C++ and to make them compatible with the **boost library of graphs**, that is, a repository for free portable C++ source libraries which work well with the C++ standard library, see `http://www.boost.org/`.

1.2 Methodologies

Exact methods. Development of algorithms for the recognition of products of graphs. Algorithms for the Cartesian, the strong and the direct product of graphs are collected in Imrich, Klavžar & Hammack [1] and in several papers e.g. by AP Imrich et al. [8, 9]. Many of them have been implemented in diploma theses at the University of Leoben, but a unified treatment and preparation that allows them to be entered into the boost library will be useful.

Approximate methods. For the strong product Hellmuth Marc, see Hellmuth, Imrich, Stadler and Klöckl [2, 5], has developed satisfactory algorithms and has implemented them for use in the boost library. The algorithms depend on the fact that neighborhoods are subproducts. This also holds for the direct product and could in principle also be applied there.

For the Cartesian product nothing of the kind exists. We plan to apply an approach similar to the one for the strong product though by the use of intervals instead of neighborhoods.

Heuristic methods. For the recognition of near products entirely new methods are needed. We expect to be able to use the fact that layers with respect to the Cartesian product are convex and the numbers of paths between vertices to make an intelligent guess on how to decompose the given graph. For the strong product similar methods may work, but here the problem is more complex.

Spectral methods. Another way to obtain a rough estimate of how to partition the vertex set may use eigenvectors of the Laplacian matrix of a graph. This feasibility of such methods will be investigated by PI Leydold and PI Biyikoglu.

Statistical methods. In order to make a suggestion which product to use we plan to rely on statistical methods. We wish to use the degree distribution, betweenness, diameter and other properties for the selection, see Leskovec et al., Kronecker graphs,
`http://arxiv.org/PS_cache/arxiv/pdf/0812/0812.4905v2.pdf`
Here too PI Leydold will use his expertise.

Quality control. Again statistical methods will be needed, we may also use entropy considerations to choose between different representations. Again PI Leydold will join in.

2 Information on funding

Research is funded by the FWF (Fonds zur Förderung der wissenschaftlichen Forschung) Austrian Science Fund.

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